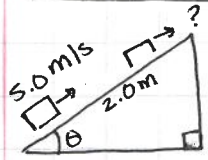
Ch. 2. & Ch. 3: Vectors & Coord. Systems. February 12, 2019

PROBLEM 2.30



$\theta = \sin^{-1}\left(\frac{1.0m}{2.0m}\right) = 30°$

$a_x = g\sin\theta$

$= 9.80\ m/s^2\left(\frac{1.0m}{2.0m}\right)$

$= 4.90\ m/s^2$ ← down the incline

x_0: 0m

x: 2.0m

v_{ox}: 5.0 m/s

v_x: ?

a_x: $-4.90\ m/s^2$

t: ?

$v_x^2 = v_{ox}^2 + 2a_x(x - x_0)$ ($x_0 = 0$)

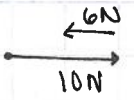
$v_x^2 = v_{ox} + 2a_x(x)$

$v_x = \pm\sqrt{v_{ox}^2 + 2a_x(x)}$

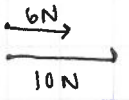
$v_x = \sqrt{(5.0\ m/s)^2 + 2(-4.90\ m/s^2)(2.0m)}$

$v_x = 2.3\ m/s$

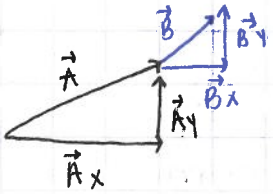
Chapter 3: Vectors & Coordinate Systems



Sum = 4N →



Sum = 16N →



Sum $\vec{C}$

$C_x = A_x + B_x$

$C_y = A_y + B_y$

February 14, 2019

Vector → physical quantity with both **magnitude** and a **direction**

↳ indicates direction in one-dimension

↓ numbers + units

(θ measured wrt + x-axis unless indicated otherwise)

$\vec{A}$: vector $\vec{A}$

A or A : Magnitude of vector $\vec{A}$

ECTOR Addition

raphically → tip-to-tail

Analytically → SOHCAHTOA or SOCA

dding Vectors Analytically

Break each vector up into x- and y-components

SOCA → x or y-component = ± (magnitude of Vector × Sin or cos → adj. side. ↳ opp. side)

must add manually

Add all the x-components together
Add all the y-components together

$$S_x = A_x + B_x + C_x$$
$$S_y = A_y + B_y + C_y$$

Find magnitude & direction of the sum:

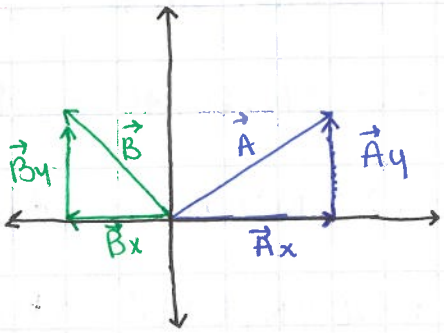
$$S = \sqrt{S_x^2 + S_y^2}$$

$$\theta = \tan^{-1}\left(\frac{S_y}{S_x}\right)$$

Check that θ is in the correct quadrant; if not

Ch. 3 February 14, 2019

Vector Components ⇒ How much a vector points along an axis (generally x- and y-axes)



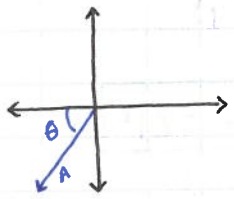
II	I
III	IV

Quadrant	x-comp.	y-component
I	+	+
II	−	+
III	−	−
IV	+	−

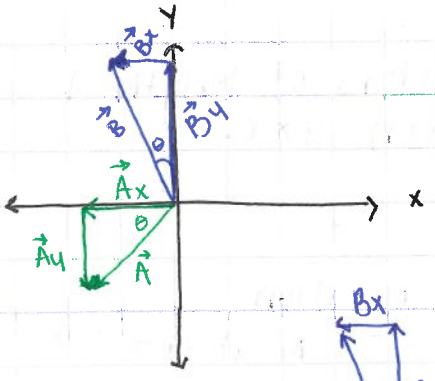
iClicker Q's

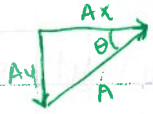
- What is the correct formula for A_y, the y-component of the vector A?

Answer: $-A\sin\theta$



Ex



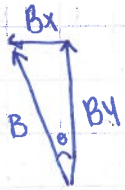


$\sin\theta = \frac{A_y}{A} \rightarrow A_y = A\sin\theta$

$\boxed{A_y = -A\sin\theta}$

$\cos\theta = \frac{A_x}{A} \rightarrow A_x = A\cos\theta$

$\boxed{A_x = -A\cos\theta}$

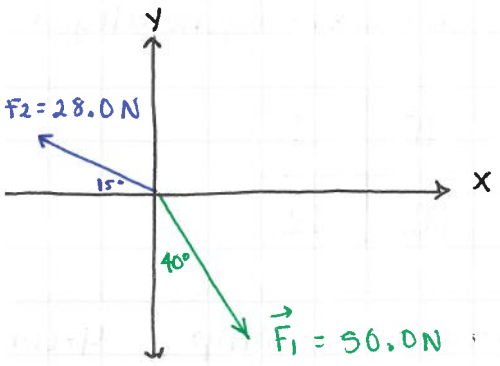


$\sin\theta = \frac{B_x}{B} \rightarrow B_x = B\sin\theta$

$\boxed{B_x = -B\sin\theta}$

$\cos\theta = \frac{B_y}{B} \rightarrow \boxed{B_y = B\cos\theta}$

, x



$F_{1x} = +(50.0N)\sin 40° = 32.1N$

$F_{1y} = -(50.0N)\cos 40° = -38.3N$

$F_{2x} = -(28.0N)\cos 15° = -27.0N$

$F_{2y} = +(28.0N)\sin 15° = +7.25N$

let $\vec{S} = \vec{F}_1 + \vec{F}_2 \Rightarrow$ $S_x = F_{1x} + F_{2x} = 32.1N - 27.0N = 5.1N$

$S_y = F_{1y} + F_{2y} = -38.3N + 7.25N = -31N$

$|\vec{S}| = \sqrt{S_x^2 + S_y^2} = \sqrt{(5.1N)^2 + (31N)^2} = \boxed{31.5N}$

$\theta = \tan^{-1}\left(\frac{S_y}{S_x}\right) = \tan^{-1}\left(\frac{-31N}{5.1N}\right) \Rightarrow \boxed{\theta = -81°}$

UNIT VECTORS

↳ dimensionless (NO unit) vectors of length 1 that point along a given axes

$\hat{i}$ = x-axis

$\hat{j}$ = y-axis

$\hat{k}$ = z-axis

Notation

$\vec{F}_1 = 50.0N$ at $\theta = -50°$

$F_{1x} = -27.0N$

$F_{2x} = 7.25N$

$\vec{F} = (-27.0N)\hat{i} + (7.25N)\hat{j}$

Ch. 3 + RECAP

February 19, 2019

RECAP

$\vec{A} = A_x\hat{i} + A_y\hat{j}$

→ Vectors can be expressed by giving a magnitude & direction ($|\vec{A}|$ and θ) or by giving the x- and y-components

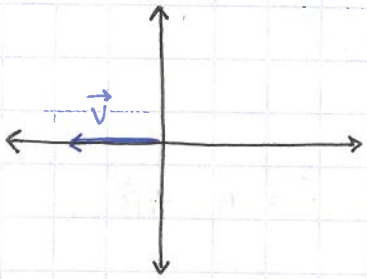
→ Vectors can not be added together like scalars

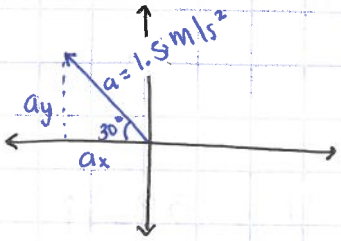
Adding Vectors Analytically

1) Break each vector up into x- and y-components (SOCA)
   x- or y- component = ± (magnitude) x sin or cos
2) Add the x-components together ($C_x = A_x + B_x$)
   Add the y-components together ($C_y = A_y + B_y$)
3) Find the magnitude & direction of the sum:
   $|\vec{C}| = \sqrt{C_x^2 + C_y^2}$ $\theta = \tan^{-1}\left(\frac{C_y}{C_x}\right)$
4) Make sure angle is correct (if x-component < 0, you must add 180° to calculator answer).

PROBLEM 3.7

$\vec{v} = (-3.5\ m/s)\hat{i}$
$= (3.5\ m/s)(-\hat{i})$





$\vec{a} = -(1.5\ m/s^2)\cos 30\,\hat{i} + (1.5\ m/s^2)\sin 30°\,\hat{j}$

$\vec{a} = (-1.30\ m/s^2)\hat{i} + (0.75\ m/s^2)\hat{j}$

Ch.3

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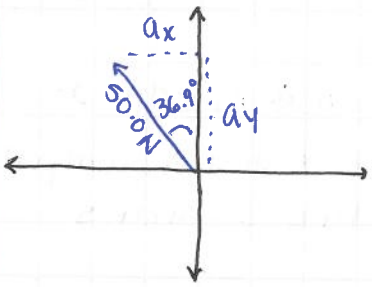
Problem 3.7 continued



$\vec{F} = -(50.0N)\sin 36.9°\,\hat{i} + (50.0N)\cos 36.9°\,\hat{j}$

$\boxed{\vec{F} = (-30N)\hat{i} + (40N)\hat{j}}$

PROBLEM 3.10

$\vec{r} = (-2.0\hat{i} - 1.0\hat{j})\,cm$

$\vec{r} = (-2.0cm)\hat{i} - (1.0cm)\hat{j}$

$r_x = -2.0cm$

$r_y = -1.0cm$

$r = \sqrt{r_x^2 + r_y^2} = \sqrt{(-2.0cm)^2 + (-1.0cm)^2} = \boxed{2.2cm}$

$\theta = \tan^{-1}\left(\frac{r_y}{r_x}\right) = \tan^{-1}\left(\frac{-1.0cm}{-2.0cm}\right) = 26.6°$

wrong quadrant so add 180°

$\boxed{\theta = 207°}$

$\vec{A} = A_x\hat{i} + A_y\hat{j}$

$\vec{B} = B_x\hat{i} + B_y\hat{j}$

- $\vec{C} = \vec{A} - \vec{B} = (A_x\hat{i} + A_y\hat{j}) - (B_x\hat{i} + B_y\hat{j})$

$= (A_x - B_x)\hat{i} + (A_y - B_y)\hat{j}$

- $\vec{D} = 2\vec{A} - 3\vec{B} = (2A_x - 3B_x)\hat{i} + (2A_y - 3B_y)\hat{j}$

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PROBLEM 3.16

Let $\vec{A} = 4\hat{\imath} - 2\hat{\jmath}$, $B = -3\hat{\imath} + 5\hat{\jmath}$, and $\vec{F} = \vec{A} - 4\vec{B}$

$$\vec{F} = \vec{A} - 4\vec{B}$$

$$\underbrace{(4\hat{\imath} - 2\hat{\jmath})}_{\vec{A}} - 4\underbrace{(-3\hat{\imath} + 5\hat{\jmath})}_{\vec{B}}$$

$$= (4 - 4(-3))\hat{\imath} + (-2 - 4(5))\hat{\jmath}$$

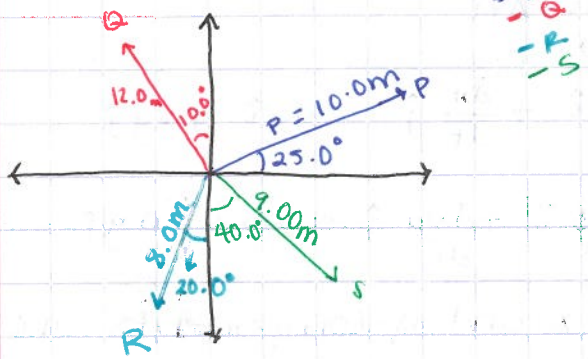
$$= 16\hat{\imath} - 22\hat{\jmath}$$

1Q

The vector A has components $A_x = +5m$ and $A_y = -10m$
Vector B has components $B_x = 45$ and $B_y = +20m$.
What is the magnitude of A - B?

(c) 50m

PROBLEM B



$-P$
$-Q$
$-R$
$-S$

$$\vec{P} = +(10.0\,m \cos 25.0°)\hat{\imath} + (10.0\,m \sin 25.0°)\hat{\jmath}$$

$$\vec{Q} = -(12.0\,m \sin 10.0°)\hat{\imath} + (12.0\,m \cos 10.0°)\hat{\jmath}$$

$$\vec{R} = -(8.0\,m \sin 20.0°)\hat{\imath} + ((8.0\,m)\cos 20.0°)\hat{\jmath}$$

$$\vec{S} = +(9.0\,m \sin 40.0°)\hat{\imath} - (9.00\,m \cos 40.0°)\hat{\jmath}$$

$$A = \vec{P} + \vec{Q} + \vec{R} + \vec{S}$$

$$A_x = P_x + Q_x + R_x + S_x = +10.0m$$

$$A_y = P_y + Q_y + R_y + S_y = +1.63m$$

$$\vec{A} = (10.0m)\hat{\imath} + (1.63m)\hat{\jmath}$$

$$|\vec{A}| = 10.2m$$

$$\theta = 9.25°$$